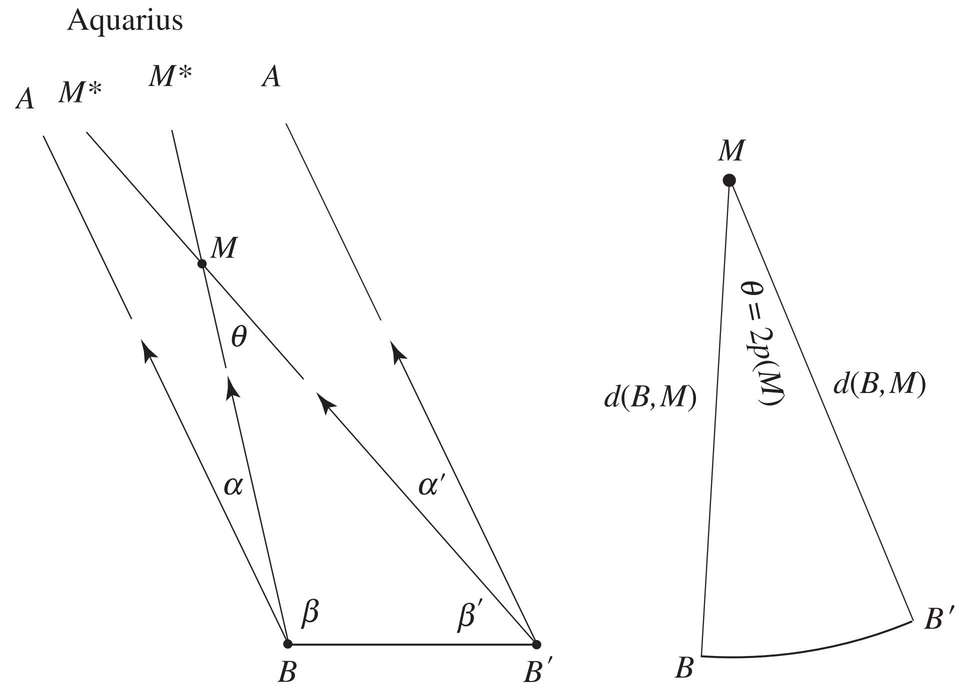
Exam A (Part I) **Name**

1. The astronomer Flamsteed observed Mars from his observatory in Derby, England in early October of the year 1672. On a clear night with his observatory in location B he observed Mars M in the night sky as the point of light M^* near a very distant star A in the constellation Aquarius and measured the angle α. A little over six hours later, after B had rotated on Earth's surface to its new position B', he did the same thing once more. The written record of his observations must



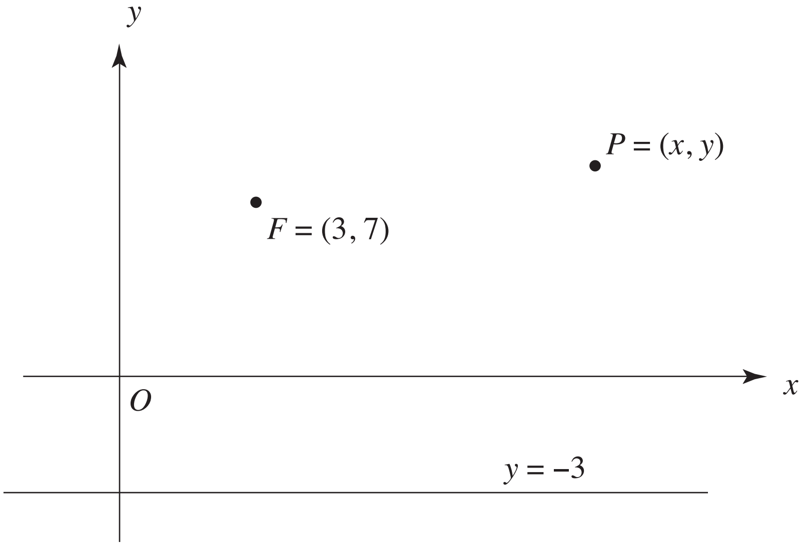
(a) (b)

have contained some version of the information provided by the two diagrams above.

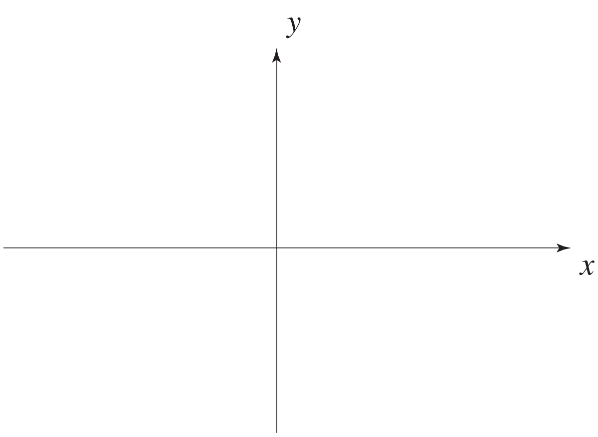
1a. Explain how Flamsteed obtained an estimate for the angle θ.

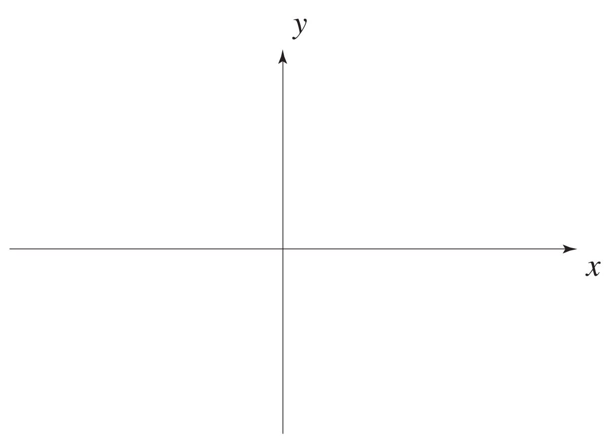
1b. With θ in hand, explain how he could determine an estimate for the distance from Earth to Mars (at the time of his observations).

2. In the coordinate plane below, the horizontal line $y = -3$ and the point $F = (3, 7)$ are given. Let $P = (x, y)$ be any point in the plane. Determine a condition on the coordinates x and y that will guarantee that P is on the parabola with focal point F and directrix $y = -3$. Is the point $(10, 9)$ on the parabola?



3. Consider the ellipse $\frac{x^2}{7^2} + \frac{y^2}{5^2} = 1$. Sketch the ellipse in the xy-plane on the left below. Express the set of points *inside* the ellipse in terms of an inequality and explain why your inequality is correct. 3b) Is the point $(-4.5, 3.8)$ inside or outside the ellipse and why?

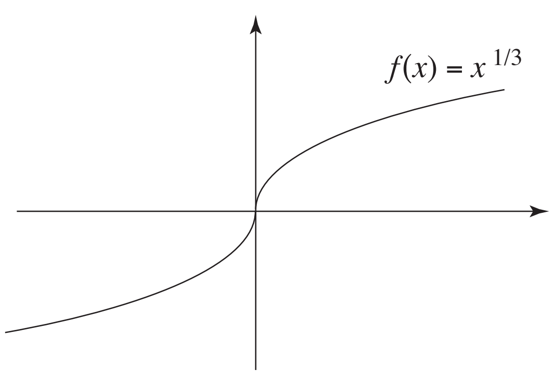




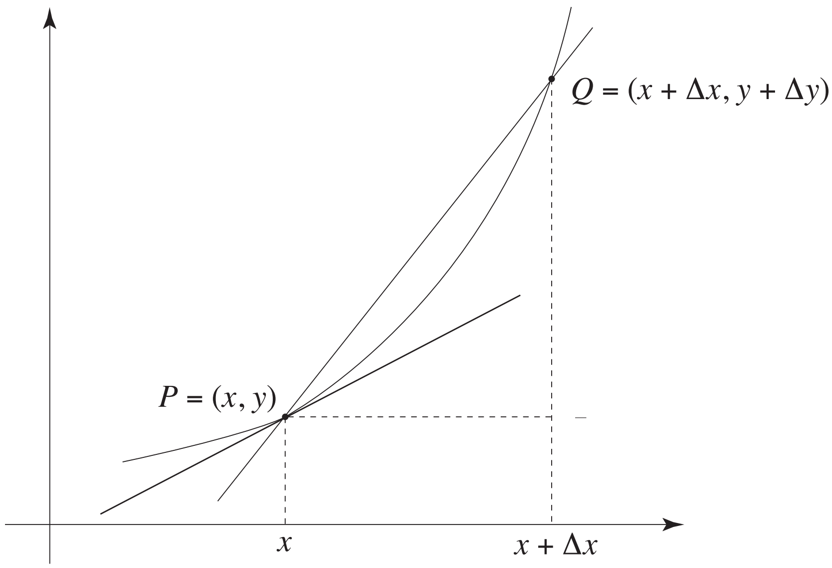
4. The equation $16x^2 + 96x + 4y^2 + 128 = 0$ represents an ellipse. Find its semimajor and semiminor axes and sketch the ellipse in the coordinate plane of the previous page on the right.

5. Describe and explain the steps required in the definition of $\int_a^b f(x)\,dx$. Do so completely in the abstract without referring to areas, volumes, rectangles, or antiderivatives.

6. Find the volume of the solid obtained by revolving the graph of the function $f(x) = x^{\frac{1}{3}}$ from $x = -1$ to $x = 8$ one revolution around the x-axis.

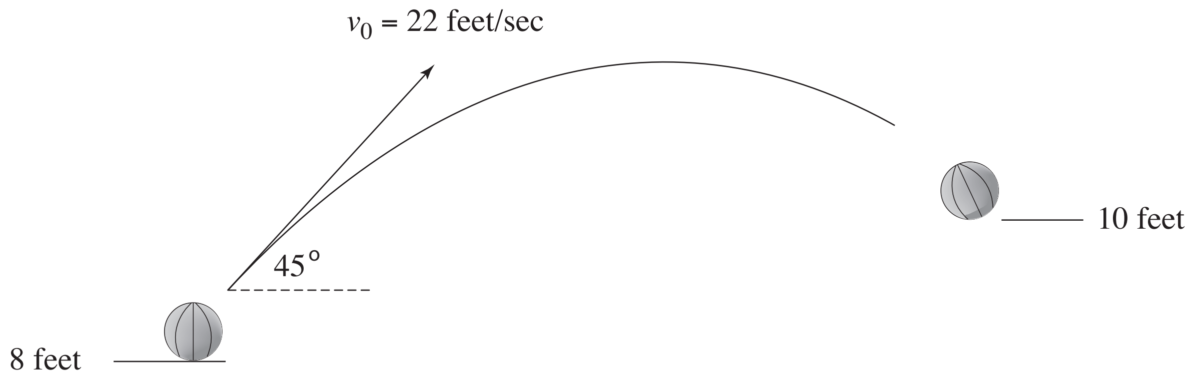


7. Use one of Newton's arguments to show that the derivative of the function $y = x^{\frac{4}{3}}$ is equal to $f'(x) = \frac{4}{3}x^{\frac{1}{3}}$. The figure below can serve as a guide.



8. The power series $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + \ldots$ converges for all x with $|x| < 1$. Use it to approximate the integral $\int_0^{\frac{3}{4}} \frac{1}{1+x^2}\, dx$ to within three decimal place accuracy. Carry six decimal places and then round off to three decimal places at the end.

9. Suppose that a basketball player's jump shot is most accurate when she releases the ball from 8 feet above the floor at an angle of 45° and a velocity of 22 feet per second. See the figure below. (The rim of the basket is 10 feet above the floor.) How far from the basket should she be taking her shots? To organize your thoughts, it may be helpful to place an xy-coordinate system into the figure above.



Relevant Formulas: $p_{\text{sec}} = \frac{3.26}{D_{\text{LY}}}$, $V = \displaystyle\int_a^b \pi f(x)^2\,dx$, $g = 32$ feet/sec^2, $\cos 45^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$.

$x'(t) = v_0 \cos\varphi_0, x(t) = (v_0\cos\varphi_0)t, y'(t) = -gt + v_0\sin\varphi_0, y(t) = -\frac{g}{2}t^2 + (v_0\sin\varphi_0)t + y_0$

$y(t) = \left(\frac{-g}{2v_0^2\cos^2\varphi_0}\right)x(t)^2 + (\tan\varphi_0)x(t) + y_0,\ t_1 = \frac{v_0\sin\varphi_0}{g}, /y(t_1) = \frac{1}{2g}v_0^2\sin^2\varphi_0 + y_0,$

$t_{\text{imp}} = \frac{1}{g}\left(v_0\sin\varphi_0 + \sqrt{v_0^2\sin^2\varphi_0 + 2gy_0}\right)$ $R = \frac{v_0}{g}\cos\varphi_0\left(v_0\sin\varphi_0 + \sqrt{v_0^2\sin^2\varphi_0 + 2gy_0}\right)$

$R_{\max} = \frac{v_0^2}{2g} + \frac{v_0}{g}\sqrt{\frac{v_0^2}{4}} = \frac{v_0^2}{2g} + \frac{v_0^2}{2g} = \frac{v_0^2}{g}$, $v(t) = \sqrt{v_0^2 + g^2t^2 - 2g(v_0\sin\varphi_0)t}$